

Current and Shot Noise in a Quantum Dot Coupled to Ferromagnetic Leads in the Large U Limit

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Using the Keldysh nonequilibrium Green function technique, we study the current and shot noise spectroscopy of a single interacting quantum dot coupled to two ferromagnetic leads with different polarizations. The polarizations of leads can be both parallel and antiparallel alignments. General formulas of current and shot noise are obtained, which can be applied in both the parallel and antiparallel alignment cases. We show that for large polarization value, the differential conductance and shot noise are completely different for spin up and spin down configurations in the parallel alignment case. However, the differential conductance and shot noise have the similar properties for parallel alignment case in the small polarization value and for antiparallel alignment case in any polarization value.

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Spin-related phenomena in semiconductor quantum dot have attracted great interest recently as they are the crucial ingredient in the emerging field of spintronics¹ and several quantum computation scheme². In addition to their potential industrial applications, these devices also provide an ideal test ground for the study of basic physics including many-body effect, such as the Kondo effect³. In this Letter, we use the Keldysh nonequilibrium Green function technique to study the current and shot noise through an interacting dot coupled to two ferromagnetic (FM) leads as a function of the applied bias voltage for parallel (P) and antiparallel (AP) lead-polarization alignments. Using the equation-of-motion approach, we obtain a general formula of current and shot noise for interacting quantum dot, which can be applied in studying the transport phenomena of dot coupled to FM leads with both the P and the AP alignments. Our results show that both the differential conductance and shot noise show the completely different behavior for spin up and spin down configurations in P alignment case with large polarization value. However, the differential conductance and shot noise show similar behavior for P alignment in small polarization value and for AP alignment in any polarization value.

The system Hamiltonian is written as

$$H = H_L + H_R + H_D + H_T. \quad (1)$$

The Hamiltonian for electrons in the left and right non-interacting metallic leads is

$$H_L + H_R = \sum_{k \in L, R; \sigma} \epsilon_{k\sigma} c_{k\sigma}^\dagger c_{k\sigma}, \quad (2)$$

where the electron creation (annihilation) operators in the leads are denoted by $c_{k\sigma}^\dagger$ ($c_{k\sigma}$). The Hamiltonian of the dot is

$$H_D = \sum_{\sigma} [\epsilon_{d\sigma} d_{\sigma}^\dagger d_{\sigma} + \frac{U}{2} n_{d,\sigma} n_{d,\bar{\sigma}}], \quad (3)$$

where d_{α}^\dagger (d_{α}) are the creation (annihilation) operators of dot electrons, and $\epsilon_{d\sigma}$ is the resonance level of the dot which can be tuned by magnetic field. The coupling of the dot to the leads is

$$H_T = \sum_{k \in L, R; \sigma, \sigma'} [V_{k\sigma, \sigma'} c_{k\sigma}^\dagger d_{\sigma'} + H.c.], \quad (4)$$

where the tunneling matrix elements $V_{k\sigma, \alpha}$ transfer electrons through an insulating barrier out of the dot.

Using the Keldysh nonequilibrium Green function formalism^{4,5}, the terminal current is given by^{6,7}

$$I = \frac{ie}{4\pi} \int d\epsilon \{ \text{Tr}[(f_L \Gamma^L - f_R \Gamma^R)(\mathbf{G}^r - \mathbf{G}^a)] + \text{Tr}[(\Gamma^L - \Gamma^R)\mathbf{G}^<] \}, \quad (5)$$

and the spectral density of shot noise in the zero-frequency is given by⁸

$$S(\omega \rightarrow 0) = \frac{e^2}{2\pi} \int d\epsilon \{ -f_L(1-f_L)(\text{Tr}[(\Gamma^L \mathbf{G}^r)^2] + \text{Tr}[(\Gamma^L \mathbf{G}^a)^2]) + if_L \text{Tr}[\Gamma^L \mathbf{G}^>] - i(1-f_L) \text{Tr}[\Gamma^L \mathbf{G}^<] + f_L \text{Tr}[\Gamma^L \mathbf{G}^> \Gamma^L (\mathbf{G}^r - \mathbf{G}^a)] + \text{Tr}[\Gamma^L \mathbf{G}^> \Gamma^L \mathbf{G}^<] - (1-f_L) \text{Tr}[\Gamma^L (\mathbf{G}^r - \mathbf{G}^a) \Gamma^L \mathbf{G}^<] \}, \quad (6)$$

where $f_{L(R)}$ are the Fermi distribution function of the left and right leads, which has different chemical potential upon a voltage bias $\mu_L - \mu_R = eV$. The coupling of the dot to the leads is characterized by the parameter

$$\Gamma_{\sigma\sigma'}^{L(R)} = 2\pi \sum_{\sigma''} \rho_{L(R), \sigma''}(\epsilon) V_{\sigma'', \sigma}^*(\epsilon) V_{\sigma'', \sigma'}(\epsilon) \quad (7)$$

with $\rho_{L(R), \sigma}$ the spin- σ band density of states in the two leads. $G_{\sigma\sigma'}^{r(a)}$ and $G_{\sigma\sigma'}^{<(>)}$ are the Fourier transform of the dot electron retarded (advanced) Green function $G_{\sigma\sigma'}^{r(a)}(t, t') = \mp i\theta(\pm t \mp t') \langle \{d_{\sigma}(t), d_{\sigma'}^\dagger(t')\} \rangle$, the lesser Green function $G_{\sigma\sigma'}^{<}(t, t') = i \langle d_{\sigma'}^\dagger(t') d_{\sigma}(t) \rangle$, and the

greater Green function $G_{\alpha\alpha'}^>(t, t') = -i\langle d_\alpha(t)d_{\alpha'}^\dagger(t') \rangle$. It is noted that Eqs. (5) and (6) expresses the current and the fluctuations of current through the quantum dot, an interacting region, in terms of the distribution functions in the leads and local properties of the quantum dot, such as the occupation and density of states.

In order to compute the current and the shot noise, one has to compute the dot electron retarded Green function G^r and Keldysh Green function $G^<$ in the presence of Coulomb interaction U . Without loss of physics we are considering here, we assume that the tunneling matrix elements are spin independent, $V_{k\sigma, \sigma'} = V_k \delta_{\sigma\sigma'}$. Using the equation-of-motion approach, we obtain the retarded dot Green function in the large U limit as:

$$G_\sigma^r(\omega) = G_{\sigma\sigma}^r(\omega) = \frac{1 - \langle n_{d,\bar{\sigma}} \rangle}{\omega - \epsilon_{d\sigma} - \Sigma_{0\sigma}^r - \Sigma_{1\sigma}^r + i0^+}. \quad (8)$$

Here

$$\begin{aligned} \Sigma_{0\sigma}^r(\omega) &= \sum_{k \in L, R} \frac{|V_k|^2}{\omega - \epsilon_{k\sigma} + i0^+} \\ &= \int \frac{d\epsilon}{2\pi} \frac{\Gamma_\sigma^L(\epsilon) + \Gamma_\sigma^R(\epsilon)}{\omega - \epsilon + i0^+}, \end{aligned} \quad (9)$$

$$\begin{aligned} \Sigma_{1\sigma}^r(\omega) &= \sum_{k \in L, R} \frac{|V_k|^2 f_{L/R}(\epsilon_{k\bar{\sigma}})}{\omega - \epsilon_{d\sigma} + \epsilon_{d\bar{\sigma}} - \epsilon_{k\bar{\sigma}} + i/2\tau_{\bar{\sigma}}} \\ &= \int \frac{d\epsilon}{2\pi} \frac{\Gamma_\sigma^L(\epsilon) f_L(\epsilon) + \Gamma_\sigma^R(\epsilon) f_R(\epsilon)}{\omega - \epsilon_{d\sigma} + \epsilon_{d\bar{\sigma}} - \epsilon + i/2\tau_{\bar{\sigma}}}, \end{aligned} \quad (10)$$

where $\Gamma_\sigma^{L(R)}(\epsilon) = \Gamma_{\sigma\sigma}^{L(R)}(\epsilon)$, and the occupation number is subject to the self-consistency condition $\langle n_{d\sigma} \rangle = -i \int \frac{d\omega}{2\pi} G_\sigma^<(\omega)$. The finite life-time in Eq. (10) for finite bias voltage and magnetic field can be obtained by using the second-order perturbation theory¹⁰.

The Green function $G_{\sigma\sigma'}^{<(>)}$ cannot be obtained by directly using the above equation-of-motion approach without introducing additional assumptions. By instead applying the operational rules as given by Langreth⁹ to the Dyson equation for the contour-ordered Green function, one can show the following Keldysh equation for the lesser and greater functions⁷

$$\mathbf{G}^{<(>)}(\omega) = \mathbf{G}^r(\omega) \Sigma_T^{<(>)}(\omega) \mathbf{G}^a(\omega). \quad (11)$$

From Eq. (8), the retarded self-energy is given by

$$\Sigma_{T,\sigma}^r(\omega) = \frac{-\langle n_{d\bar{\sigma}} \rangle (\omega - \epsilon_{d\sigma}) + (\Sigma_{0\sigma}^r + \Sigma_{1\sigma}^r)}{1 - \langle n_{d\bar{\sigma}} \rangle}. \quad (12)$$

We then arrive at

$$\begin{aligned} \Sigma_{T,\sigma}^{<}(\omega) - \Sigma_{T,\sigma}^{>}(\omega) &= \Sigma_{T,\sigma}^a(\omega) - \Sigma_{T,\sigma}^r(\omega) \\ &= i(1 - \langle n_{d\bar{\sigma}} \rangle)^{-1} \{ (\Gamma_\sigma^L(\omega) + \Gamma_\sigma^R(\omega)) \\ &\quad - 2 \text{Im} \Sigma_{1\sigma}^r(\omega) \}. \end{aligned} \quad (13)$$

To determine $\Sigma_{T,\sigma}^{<}$ and $\Sigma_{T,\sigma}^{>}$, we follow the ansatz proposed in Ref.¹¹ to assume further that these self-energies have the form

$$\Sigma_{T,\sigma}^{<} = i[\Gamma_\sigma^L(\omega) f_L(\omega) + \Gamma_\sigma^R(\omega) f_R(\omega)] R_\sigma(\omega), \quad (14a)$$

$$\Sigma_{T,\sigma}^{>} = -i[\Gamma_\sigma^L(\omega)(1 - f_L(\omega)) + \Gamma_\sigma^R(\omega)(1 - f_R(\omega))] R_\sigma(\omega), \quad (14b)$$

where $R(\omega)$ can be regarded as a renormalization factor due to strong electron-electron Coulomb repulsion in the dot. A little algebra yields

$$R_\sigma(\omega) = \frac{1}{1 - \langle n_{d\bar{\sigma}} \rangle} \left\{ 1 - \frac{2}{\Gamma_\sigma} \text{Im} \Sigma_{1\sigma}^r(\omega) \right\} \quad (15)$$

where $\Gamma_\sigma = \Gamma_\sigma^L + \Gamma_\sigma^R$.

As an extension, we can assume in general the “scattering in” and “scattering out” self-energies to have the form

$$\Sigma_T^{<} = i[\mathbf{\Gamma}^L(\omega) f_L(\omega) + \mathbf{\Gamma}^R(\omega) f_R(\omega)] \mathbf{R}(\omega), \quad (16a)$$

$$\Sigma_T^{>} = -i[\mathbf{\Gamma}^L(\omega)(1 - f_L(\omega)) + \mathbf{\Gamma}^R(\omega)(1 - f_R(\omega))] \mathbf{R}(\omega), \quad (16b)$$

Eqs. (16a) and (16b) leads to

$$\begin{aligned} \mathbf{G}^r - \mathbf{G}^a &= \mathbf{G}^> - \mathbf{G}^< \\ &= -i \mathbf{G}^r (\mathbf{\Gamma}^L + \mathbf{\Gamma}^R) \mathbf{R} \mathbf{G}^a. \end{aligned} \quad (17)$$

Substitution of Eq. (17) and Eq. (11) with Eq. (16) into Eqs. (5) and (6) gives rise to

$$\begin{aligned} I &= \frac{e}{2\pi} \int d\epsilon \text{Tr} \{ \mathbf{\Gamma}^L [\mathbf{G}^< + f_L (\mathbf{G}^r - \mathbf{G}^a)] \} \\ &= \frac{e}{2\pi} \int d\epsilon \text{Tr} \{ \mathbf{\Gamma}^L [i \mathbf{G}^r (f_L \mathbf{\Gamma}^L + f_R \mathbf{\Gamma}^R) \mathbf{R} \mathbf{G}^a \\ &\quad - i f_L \mathbf{G}^r (\mathbf{\Gamma}^L + \mathbf{\Gamma}^R) \mathbf{R} \mathbf{G}^a] \} \\ &= \frac{e}{2\pi} \int d\epsilon [f_L - f_R] \text{Tr} [\mathbf{T}], \end{aligned} \quad (18)$$

and

$$\begin{aligned} S &= \frac{e^2}{2\pi} \int d\epsilon \{ [f_L(1 - f_L) \text{Tr} \{ \mathbf{T} [1 + 2(\mathbf{\Gamma}^R \mathbf{R})^{-1} (\mathbf{R} - 1)] \} \\ &\quad + f_R(1 - f_R) \text{Tr} [\mathbf{T}] + (f_L - f_R)^2 \text{Tr} [(1 - \mathbf{T}) \mathbf{T}] \} \end{aligned} \quad (19)$$

where the transmission coefficient matrix is defined as $\mathbf{T} = \mathbf{G}^a \mathbf{\Gamma}^L \mathbf{G}^r \mathbf{\Gamma}^R \mathbf{R}$. We remark that Eqs. (18) and (19) are the general formulas for current and shot noise in the interacting dots with large U , which can be applied in studying the spin-transport of both the P and AP configurations. It is noted that the formula of proportional coupling (i. e., $\Gamma_\sigma^L = \lambda \Gamma_\sigma^R$) does not work in the AP alignment case at finite bias voltage and magnetic field. However, one can easily to compute the current and shot noise for AP alignment configuration by

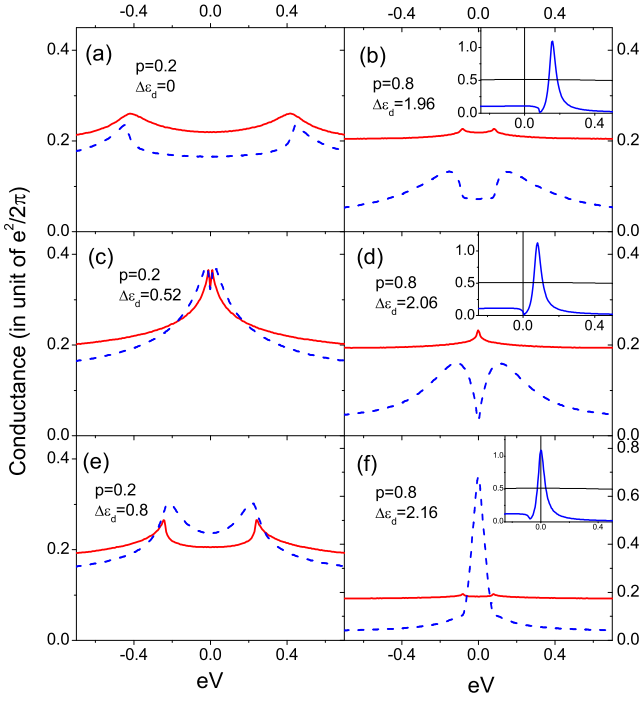


FIG. 1: The zero-temperature differential conductance of spin up (red-solid line) and spin down (blue-dashed line) for FM leads with P alignment as a function of bias voltage without (a) and with (b-f) Zeeman splitting $\Delta\epsilon_d = \epsilon_{d\uparrow} - \epsilon_{d\downarrow}$. Here $\Gamma_0 = 1$, the dot level in the absence of magnetic field is $\epsilon_{d\uparrow} = \epsilon_{d\downarrow} = -3.0$, and the band width is 100. Insets in (b,d,f) indicate the DOS of spin down configuration as a function of bias voltage near $eV = 0$.

using our formalisms Eqs. (18) and (19). It is easy to find that for the noninteracting case, the above two expressions are just the Landauer-Büttiker formalisms developed based on the scattering matrix theory. The connection between the two formalisms was first established by Meir and Wingreen for the current⁶, and by Zhu and Balatsky for the shot noise⁸.

In the wideband limit, we assume that $\Gamma_\sigma^{L(R)}$ to be energy independent, but polarization dependent with $\Gamma_\uparrow^L = \Gamma_\uparrow^R = (1+P)\Gamma_0$, $\Gamma_\downarrow^L = \Gamma_\downarrow^R = (1-P)\Gamma_0$ for P alignment, while $\Gamma_\uparrow^L = \Gamma_\downarrow^R = (1+P)\Gamma_0$, $\Gamma_\downarrow^L = \Gamma_\uparrow^R = (1-P)\Gamma_0$ for AP alignment, where Γ_0 describes the tunneling coupling between the dot and the nonmagnetic leads, and $0 \leq p < 1$ characterizes the polarization of leads. In Fig. 1 we plot the zero-temperature differential conductance of spin up (solid line) and spin down (dashed line) for FM leads with P alignment as a function of bias voltage. One interesting observation is that the behavior of differential conductance is quite different for small and large P values. For small $P = 0.2$, Fig. 1(a) shows that the zero-bias peak of differential conductance for nonmagnetic leads¹⁰ splits into two peaks even in the absence of magnetic field, and this splitting can be tuned away by applying an appropriate magnetic field [Fig. 1(c)] and can be recovered

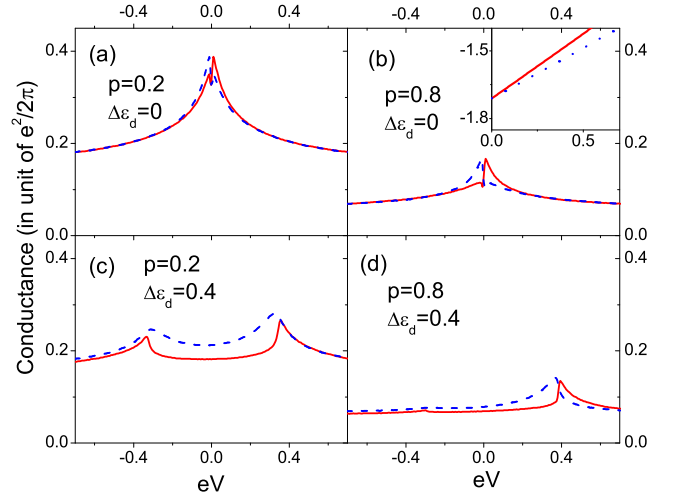


FIG. 2: The differential conductance for FM leads with AP alignment.

ered by increasing the magnetic field further [Fig. 1(e)]. This result agrees well with that in Ref. ¹², which is due to the splitting of the dot levels renormalized by the spin-dependent interacting self-energy $\tilde{\epsilon}_{d\sigma}$ satisfying the self-consistent equation $\tilde{\epsilon}_{d\sigma} = \epsilon_{d\sigma} + \text{Re} \Sigma_{1\sigma}^r(\tilde{\epsilon}_{d\sigma}, \tilde{\epsilon}_{d\bar{\sigma}})$. For large $P = 0.8$ [Figs. 2 (b,d,f)], the behavior of spin up is similar to that of small $P = 0.2$, which shows one peak at appropriate value of magnetic field and two peaks when the field is away from the appropriate value. However for the spin down configuration, the differential conductance shows a small "flat" near $eV = 0$ when the magnetic field is smaller than the appropriate value, while it shows a "dip" at $eV = 0$ when the field is at the appropriate value, and finally it shows a large maximum when the field increases further. This interesting behavior can be understood with the help of DOS for spin down near the zero bias voltage (shown as insets of Figs. 1(b,d,f)). The DOS for spin down configuration shows a dip at $\omega = \tilde{\epsilon}_{d\downarrow} - \tilde{\epsilon}_{d\uparrow}$ because $\text{Re} \Sigma_{1\downarrow}^r(\omega)$ grows logarithmically at this value, and the dip position can be tuned by magnetic field around the appropriate value corresponding to the single peak for spin up configuration. In Fig. 2 we plot the differential conductance for FM leads with AP alignment. In this case, the behavior of differential conductance looks similar for spin up and spin down configurations, which has one peak in the absence of magnetic field [Fig. 2(a,b)] and two shifted peaks in the presence of magnetic field [Fig. 2(c,d)]. The small difference between the up and down spin configurations is due to the small (compared to the P alignment case) splitting of renormalized dot levels for different spin configurations at finite voltage (as shown in the inset of Fig. 2(b)).

In Figs. 3 and 4, we plot the corresponding differential shot noise for FM leads with P and AP alignments as a function of the bias voltage. For P alignment case, the differential shot noise Fig. 3 also shows the different behavior for spin up and down configurations in large P val-

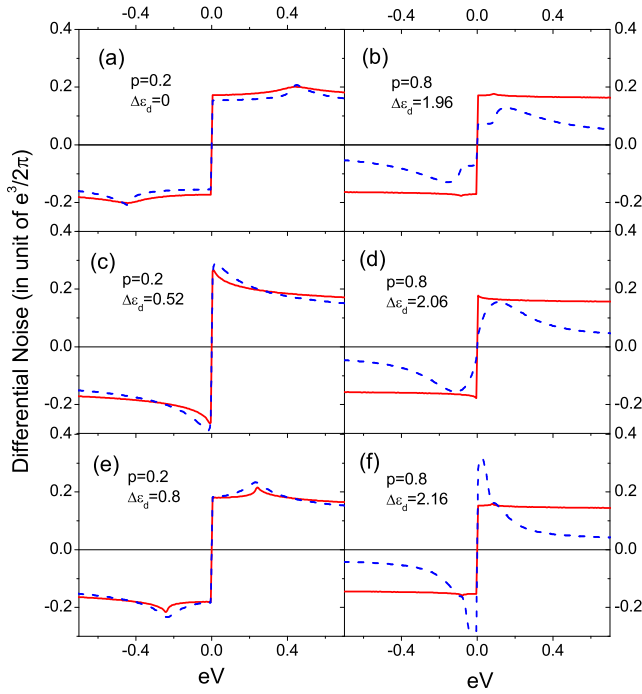


FIG. 3: The differential shot noise for FM leads with P alignment.

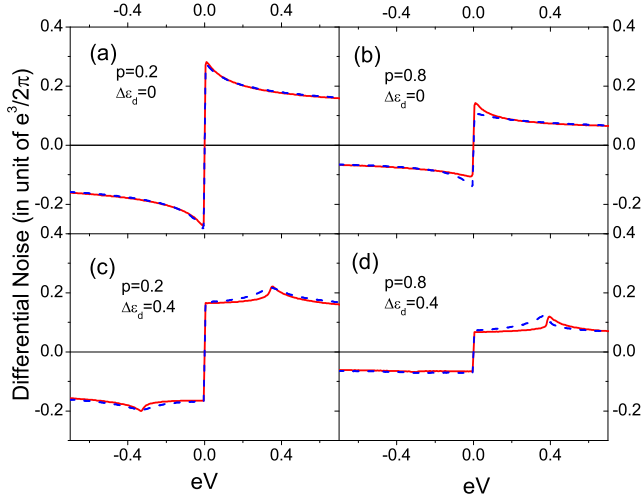


FIG. 4: The differential shot noise for FM leads with AP alignment.

ues, while similar behavior for small P value. However, for AP alignment case, the differential shot noise Fig. 4 shows the similar behavior for different spin configurations. Since the transmission probability corresponding to the conductance peaks (as shown in Figs. 1 and 2) is small, the differential noise is approximately proportional to the conductance and only shown one single peak rather than two peaks.

To summarize, using the Keldysh nonequilibrium Green function technique, we have studied the current and shot noise spectroscopy of a single dot with Coulomb interaction coupled to FM leads with P and AP polarization alignments. We have shown that the lead alignments affect both the current and current fluctuations. For large polarization value, the spin up and spin down configurations have the different behavior in the differential conductance and shot noise as a function of bias voltage in P alignment case. While the differential conductance and shot noise show similar behavior for different spin configurations in P alignment with small polarization value and in AP alignment case with any polarization value. The derived current and shot noise formulism can be applied to more complicated system. Work along this line is still in progress.

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